MODULE 1 OVERVIEW

TEKS* Addressed:
7.4A, 7.4B, 7.4C, **7.4D,** 7.4E, 7.5B, 7.6D, 7.8C, **7.9B, 7.9C**
*Bold TEKS = Readiness Standard

Thinking Proportionally

Sessions: **35**

Why is this module named *Thinking Proportionally*?

Thinking Proportionally relies heavily on students' reasoning about quantities to develop strategies and algorithms for solving problems involving ratios and proportional relationships. Throughout the module, students use reasoning about numbers in relation to each other rather than as abstract objects to analyze and describe relationships.

Students identify and describe proportional and non-proportional mathematical and real world situations, because in order to discern the characteristics of proportional relationships, students must experience relationships that are not proportional.

Developing the ability to think proportionally requires a variety of experiences and time to achieve fluency; therefore, this module includes opportunities both to reason about quantities and to develop precision and fluency with proportional relationships.

The Research Shows . . .

"The cross product is not the only way to solve proportions… This strategy, when understood, is useful when numbers are more challenging and the unit rate or scale factor is not as easy to calculate." In fact, a synthesis of research on student learning about ratio and proportion concludes, "Symbolic or mechanical methods, such as the cross-product algorithm, for solving proportions do not develop proportional reasoning and should not be introduced until students have had many experiences with intuitive and conceptual methods."

Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 6–8, Vol 3, 2nd Edition | Pages 218, 219

What is the mathematics of *Thinking Proportionally*?

Thinking Proportionally contains three topics: *Circles and Ratios, Fractional Rates,* and *Proportionality.* Students investigate special ratios, including pi and ratios of fractions, as they develop and connect formulas for the circumference and area of circles and improve their fluency with writing and interpreting unit rates. They also investigate tables, graphs, equations, and verbal descriptions of proportional relationships and use multiple representations to solve a wide variety of proportion problems.

10 SESSIONS
9 LEARNING • 1 ASSESSMENT

TOPIC 1 *Circles and Ratios*

Learning Together: 7 Sessions

TEKS: 7.4B, 7.5B, 7.6D, 7.8C, **7.9B, 7.9C**

Students use the ratio of the circumference of a circle to the length of its diameter to develop an understanding of pi.

- Students derive the formula for the circumference of a circle, which they then use to derive the formula for the area of a circle.
- Students solve real-world and mathematical problems related to the circumference and area of circles.

Learning Individually: 3 Sessions

Targeted Skills Practice for *Circles and Ratios*

- Students analyze the circumference to diameter ratio of circles.
- Students calculate the circumference and area of given circles.
- Students determine the circumference or area to solve real-world problems.
- Students determine the area of composite figures that include circles and semicircles.

8 SESSIONS
7 LEARNING • 1 ASSESSMENT

TOPIC 2 *Fractional Rates*

Learning Together: 5 Sessions

TEKS: 7.4B, **7.4D**, 7.4E

Students write and simplify unit rates, including those composed of fractions.

- Students write unit rates from a variety of contexts and then connect equivalent ratios and rates to writing and solving proportions.

Learning Individually: 2 Sessions

Targeted Skills Practice for *Fractional Rates*

- Students write and compare fractional rates.
- Students determine unit rates from tables.
- Students convert rates to different units.
- Students rewrite rates as unit rates.
- Students set up and solve proportions.

16 SESSIONS
15 LEARNING • 1 ASSESSMENT

TOPIC 3 *Proportionality*

Learning Together: 11 Sessions

TEKS: **7.4A**, 7.4C, **7.4D**

Students examine representations of various relationships and determine which are proportional.

- Students practice recognizing proportional relationships from tables, graphs, and equations.
- Students use proportions to understand the constant of proportionality (unit rate) and equations of direct variation relationships.

Learning Individually: 4 Sessions

Targeted Skills Practice for *Proportionality*

- Students determine whether given relationships are proportional.
- Students identify the constant of proportionality and represent proportional relationships using tables, equations, and graphs.

How is *Thinking Proportionally* connected to prior learning?

Thinking Proportionally builds on students' experiences with ratio and proportional relationships in Grade 6. Students previously developed an understanding of ratio concepts, including unit rate, and have used ratio reasoning to solve problems. The focus in Grade 6 was for students to reason about ratios and to develop informal strategies for determining equivalent ratios. Students used tables, strip diagrams, double number lines, and equations.

They did not use the formal method of means and extremes to solve proportions. In *Thinking Proportionally*, students develop algorithms to replace their informal strategies for solving proportion problems.

Math Representation

The local bakery sells muffins in variety packs of blueberry, pumpkin, and bran muffins. They always sell the muffins in the ratio of 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin.

Suppose you purchase an 18-pack of muffins. How many blueberry, pumpkin, and bran muffins will you purchase?

There are 6 muffins represented in the strip diagram, and you want 18 total muffins that are in the same ratio.

Therefore, to determine how many muffins you need to maintain the same ratio, you can divide 18 by 6.
18 ÷ 6 = 3

Therefore, each rectangle will represent 3 muffins.

Blueberry	3	3	3
Pumpkin	3	3
Bran	3
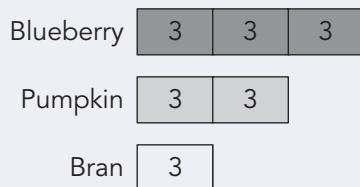
From the strip diagram, you can see that there are 9 blueberry muffins, 6 pumpkin muffins, and 3 bran muffins.

When will students use knowledge from *Thinking Proportionally* in future learning?

This module formalizes equations and graphs of proportional relationships. In **Module 3** *Reasoning Algebraically,* students will determine whether relationships are proportional and, when so, use what they know about proportional relationships to analyze and solve problems.

- To explore one of the last unknown regions on our planet, companies are starting to produce single-person, submersible deep-sea submarines like the *Deep Flight I.* Suppose the submarine *Deep Flight I* is going to do a dive starting at sea level, descending 480 feet every minute. *(The Math Representation shows a visual of this concept)*

- You can determine the depth of the submarine after 2.5 minutes using the proportional relationship. The equation representing the relationship is
$$d = -480t$$
$$d = -480(2.5) = -1200 \text{ feet}$$

In Grade 8, students will connect proportional relationships to linear equations, specifically connecting the constant of proportionality to the slope of the graph of the linear equation $y = mx$ and use similar triangles to explain that the slope of a line is constant. Students will build on equations of proportional relationships to write and analyze equations of linear non-proportional relationships as they deepen their understanding of slope.

Math Representation

Independent Quantity	Dependent Quantity
Time	Depth
minutes	feet
t	d
0	0
1	−480
2	−960
3	−1440
4	−1920
5	−2400
6	−2880
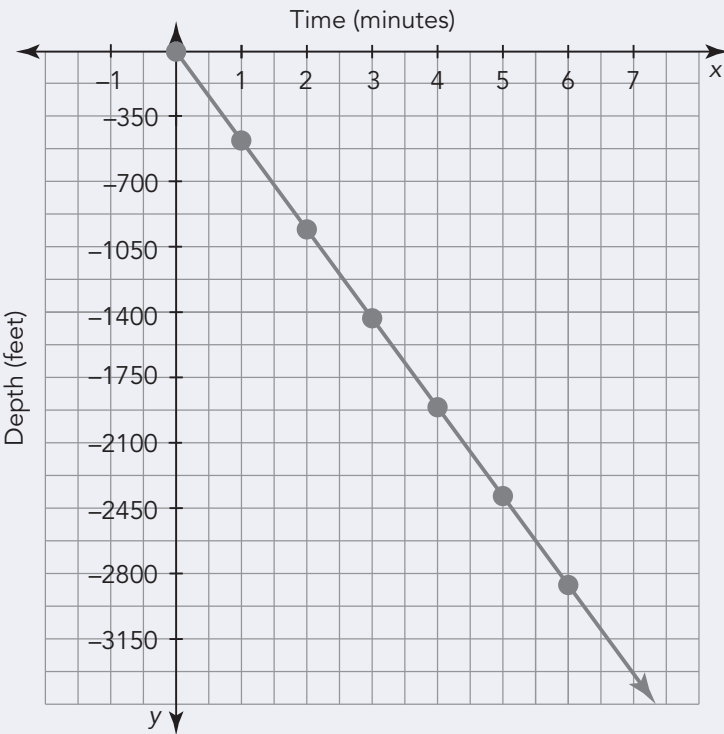
1 Thinking Proportionally

MODULE 1 Assessment Summary

Topic	Topic Title	Name	Administered	TEKS*
1	**Circles and Ratios**	End of Topic Assessment	After Topic 1	7.5B **7.9B** **7.9C**
2	**Fractional Rates**	End of Topic Assessment	After Topic 2	7.4B **7.4D** 7.4E
3	**Proportionality**	End of Topic Assessment	After Topic 3	**7.4A** 7.4C **7.4D**

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